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Influence of natural convection on the parameters of thermal explosion in the horizontal cylinder

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Abstract

The problem of a thermal explosion in the cylinder has been solved taking into account the natural convection. Both the classical solution of the Frank-Kamenetskii model and a realistic model of a thermal emission which takes into consideration reverse endothermic processes have been considered. It is shown that in this case the solution exists for any value of the energy influx into system. However, under certain conditions a sharp transition into the area of high temperatures still occurs which corresponds to the thermal explosion. It is shown that convection leads to significant change of the parameters of the thermal explosion. The results of this consideration are of interest for the theory of lasers and discharges where the Rayleigh numbers are rather large and the influence of convection appears to be essential.

1. Introduction

Historically the physics of gas lasers and discharges has been developed without taking into account natural convection processes. Even the terminology developed in this area creates an incorrect representation about observable processes. Indeed, systems without gas circulation are called lasers with diffusive cooling, and the term "convective heat dissipation" is reserved for systems with compulsory gas circulation. At the same time, simple estimates show that Rayleigh numbers in lasers and discharges governed by the convection are sufficiently large and thus it is impossible to neglect the influence of convection. At present, when optimization of the laser systems is a forefront problem, the question of how to consider the heat dissipation appears to be very important. In fact, the main obstacle in increasing of capacity of lasers with diffusive cooling is the heat dissipation: Indeed, the increase of translational temperature in the nonequilibrium area leads to abrupt increase of energy dissipation speed (thermal explosion) which results in the discharge contraction and termination of lasing.

The theory of thermal explosion applied to a nonequilibrium gas has been developed starting from the classical works of Semenov and Frank-Kamenetskii [1,2]. In these papers various conditions of the heat dissipation at the boundaries of a system have been analyzed. In the Semenov's theory the heat exchange inside the system is assumed to be much faster than an exchange with an environment, while in the Frank-Kamenetskii's theory the inverse situation has been considered. Application of the Frank-Kamenetskii's theory to nonequilibrium gas without taking into account the reverse processes in the system has been considered in Ref. [3]. In Refs. [4,5] the reverse processes have been taken into account and all range of possible system states has been considered. However in all these works the natural convection has been neglected. One notes that possible influence of convection has already been mentioned in Ref. [2], but calculations have not been made at that time. The influence of convection on thermal explosion for the cases of horizontal and vertical plane

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Nomenclature

g	acceleration of gravity
b_c	critical parameter which controls the onset of the thermal explosion with convection
b_{wc}	critical parameter which controls the explosion without convection
Pr	Prandtl number, $\mu c/\lambda$
R_o	radius of the cylinder
k	parameter describing absorption of energy in reverse endothermic processes
Ra_T	modified Rayleigh number, $\rho g \beta L^3 T_o g/\mu\alpha$
R	dimensionless radial coordinate
T_o	temperature of the cylinder
T	current temperature
U, V	velocity components in the radial and angular directions, respectively
u, v	dimensionless velocity components in the radial and angular directions, respectively
F_R, F_θ	the components of a gravity in the radial and angular directions respectively referred to the unit volume
C	thermal capacity referred to a mass unit
P	pressure
Q	internal heat generation
q	dimensionless internal heat generation

Greek symbols

λ	thermal conductivity
φ	dimensionless temperature
α	thermal diffusivity
ν	kinematic viscosity
ρ	mean density
ω	dimensionless vorticity
θ	angular coordinate
ψ	dimensionless streamfunction
δ	ratio of the critical parameter b_c to parameter b_{wc}
β	thermal volumetric expansion coefficient
μ	dynamic viscosity

layers without the reverse reactions has been considered in Refs. [6,7]. The vital difference for the cylindrical geometry is the existence of a fluid motion at any temperature differences and the influence of this motion on heat transfer.

The convective structure of fluxes depends on the geometry of the object.

The problem of coaxial cylinders which is important for technical applications or the problems of the heat transmission under the condition of a non-uniform heating of the cylinder walls has usually been considered for the case of the horizontal cylinder. Both for the system of coaxial cylinders, and for a horizontal cylinder the influence of convection in the case of volumetric heat generation has already been considered in [8] using the model of constant energy injection. In this work it has been shown that natural convection largely determines the formation of the temperature distribution. The properties of this distribution may explain failures in the envelopment of coaxial lasers because the existence of the additional cylinder at the center prevents the convection. The problem of a heat generation which depends on the external temperature is also of practical interest. Such processes occur in chemical reactors, lasers and discharges. In this case, in parallel with the natural convection, there can be a thermal explosion. The aim of the paper is to study the influence of convection on the parameters of the thermal explosion. The analysis is carried out within the framework of Boussinesq approximation which is correct if the Rayleigh numbers are not too large (this condition is obviously satisfied for the values of $Ra_T \leqslant 5–10 \times 10^3$, used as a result). Taking into account both direct, and reverse processes it is possible to consider both the low-temperature and the high-temperature modes.

2. Analysis

The configuration to be studied and the coordinate system are shown in Fig. 1. The fluid is contained inside the cylinder of radius R_o, which is held at temperatures T_o. The uniform heat generation Q is assumed to be spatially homogeneous. Density change in the fluid is neglected everywhere except for the buoyancy, and all other physical properties of the fluid are assumed to be constant (Boussinesq approximation).

We consider a two-dimensional problem, and use the cylindrical coordinates (R, θ), where the angle θ is measured counter-clockwise with respect to the upward vertical axis which contains the center of the cylinders (Fig. 1).

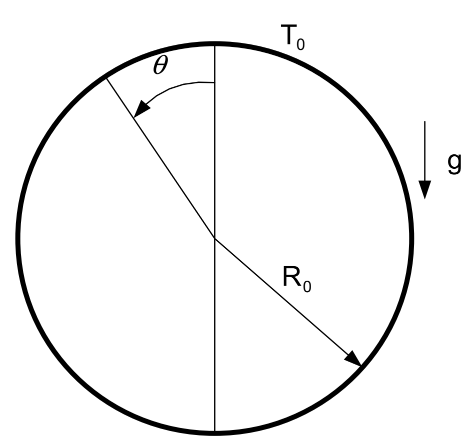

Fig. 1. The horizontal cylinder. T_0 – temperature on a surface of the cylinder, R_0 – radius of the cylinder, g – acceleration of gravity.

In the Boussinesq approximation the dimensional governing equations are expressed as

$$\frac{\partial U}{\partial R}+\frac{U}{R}+\frac{1}{R}\frac{\partial V}{\partial \theta}=0, \tag{1}$$

$$\rho\left[\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial R}+\frac{V}{R}\frac{\partial U}{\partial \theta}+\frac{V^2}{R}\right] = -\frac{\partial P}{\partial R}+\mu\left[\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}+\frac{1}{R^2}\frac{\partial^2 U}{\partial \theta^2}-\frac{U}{R^2}-\frac{2}{R^2}\frac{\partial V}{\partial \theta}\right]+F_R, \tag{2}$$

$$\rho\left[\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial R}+\frac{V}{R}\frac{\partial V}{\partial \theta}+\frac{VU}{R}\right] = -\frac{1}{R}\frac{\partial P}{\partial \theta}+\mu\left[\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}+\frac{1}{R^2}\frac{\partial^2 V}{\partial \theta^2}-\frac{V}{R^2}+\frac{2}{R^2}\frac{\partial U}{\partial \theta}\right]+F_\theta, \tag{3}$$

$$\rho c\left[U\frac{\partial T}{\partial R}+\frac{V}{R}\frac{\partial T}{\partial \theta}+\frac{\partial T}{\partial t}\right]=\lambda\left[\frac{\partial^2 T}{\partial R^2}+\frac{1}{R}\frac{\partial T}{\partial R}+\frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2}\right]+Q, \tag{4}$$

where U and V are the radial and angular velocities, respectively, P is the pressure, T is the temperature, F_R and F_θ are the components of a gravity per unit volume, ρ is a density, μ is the dynamic viscosity, $\boldsymbol{c}$ is the thermal capacity per unit of mass and λ is the thermal conductivity.

The temperature constituent of the body-force terms can be written as functions of the temperature difference:

$$F_R = g\rho\beta(T-T_o)\cos\theta, \tag{5}$$

$$F_\theta = g\rho\beta(T-T_o)\sin\theta, \tag{6}$$

where $T(R,t)$ is the current temperature of a liquid and β is the factor of thermal expansion.

One can introduce the stream function Ψ which satisfies the continuity equation by setting:

$$U=R^{-1}\partial\Psi/\partial\theta, \quad V=-\partial\Psi/\partial R. \tag{7}$$

Here the dimensionless parameters are

$$\psi=\frac{\Psi}{\alpha}, \quad r=\frac{R}{R_o}, \quad \varphi=\frac{T-T_o}{T_o q}, \quad u=\frac{UR}{\alpha}, \quad v=\frac{VR}{\alpha}, \quad q=\frac{QR^2}{\lambda T_o}, \tag{8}$$

where $\alpha=\lambda/\rho c$ is the thermal diffusivity.

The resulting equations can be simplified by introducing the vorticity ω, defined as

$$\omega=-\nabla^2\psi, \tag{9}$$

where $$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}, \tag{10}$$

is the Laplacian in cylindrical coordinates.

Different models of heat generation lead to different physical consequences. Moreover, some effects observed in real systems may disappear as a result of the simplifications made in some models. Firstly, let us express the heat generation in the form, used by Frank-Kamenetskii [2]. In this case heat generation is expressed as $Q=Q_0\exp(A\cdot T)$.

Now the initial system (1)–(4) is reduced to the dimensionless governing system:

$$\nabla^2\psi=-\omega \tag{11}$$

$$\nabla^2\omega=\frac{1}{Pr}\left[\frac{\partial\omega}{\partial t}+u\frac{\partial\omega}{\partial r}+\frac{v}{r}\frac{\partial\omega}{\partial\theta}\right] + Ra_T\left[\sin\theta\frac{\partial\varphi}{\partial r}+\frac{1}{r}\cos\theta\frac{\partial\varphi}{\partial\theta}\right] \tag{12}$$

$$\nabla^2\varphi=u\frac{\partial\varphi}{\partial r}+\frac{v}{r}\frac{\partial\varphi}{\partial\theta}-\exp(b_c\varphi)+\frac{\partial\varphi}{\partial t}, \tag{13}$$

where $Pr=\mu c/\lambda$ is the Prandtl number and $Ra_T=\rho g\beta L^3 T_o q/\mu\alpha$ is the modified Rayleigh number.

In the present problem the boundary conditions correspond to two impermeable isothermal walls of the cylinders which has constant radii and one vertical symmetry axis at $\theta=0$ and $\theta=180°$. The stream function is equal to zero at all boundaries along both walls as well as along the symmetry axis as there are no fluxes through the walls and through the plane. Then the angular derivative of the temperature and vorticity disappear along the symmetry line.

$$\omega=-\partial^2\psi/\partial r^2. \tag{14}$$

The boundary conditions in the symmetry plane become

$$\psi=\omega=\partial\varphi/\partial\theta=0, \tag{15}$$

while at a wall of the cylinder one obtains

$$\psi=u=v=0, \quad \omega=-\partial^2\psi/\partial r^2, \quad \varphi|_{r=ro}=0. \tag{16}$$

In the absence of convection equation (13) can be written as

$$\nabla^2\varphi=-\exp(b_{wc}\cdot\varphi) \tag{17}$$

with boundary conditions (15,16).

Eq. (17) is well-known in the Frank-Kamenetskii thermal explosion theory.

3. Results and discussion

One notes that the reverse processes have not been taken into account in the consideration presented above. In this case the appearance of the thermal explosion corresponds to the disappearance of the solution. One may introduce a dimensionless parameter δ determined as the ratio of the critical parameter b_c which controls the onset of the thermal explosion with convection, to parameter b_{wc} which controls the explosion without convection ($\delta=b_c/b_{wc}$). The dependence of δ on the modified Rayleigh number Ra_T is presented in Fig. 2. The figure shows, that at small values of Ra_T the value of δ is of the order of 1, i.e. b_c fully agrees with the results obtained by Frank-Kamenetskii. However, the role of convection increases with the increasing Ra_T, leading simultaneously to the improvement of cooling in

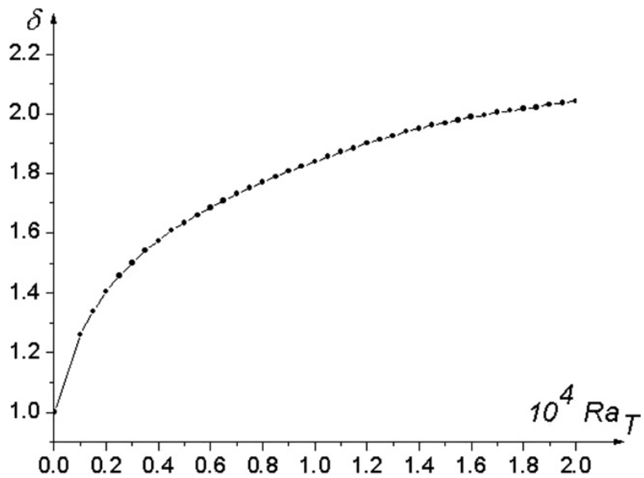

Fig. 2. Parameter δ ($\delta = b_c/b_{wc}$) as a function of the modified Rayleigh number Ra_T.

the system, and as a result the thermal explosion occurs much later.

Reverse processes can be taken into account if the heat generations expressed as $Q = Q_0 \exp(-A/T) \cdot \left(1 - \frac{kT}{T_0}\right)$, where k is the parameter describing absorption of energy in reverse endothermic processes.

The account of reverse processes results in a change of only the last equation in the system (11)–(13) which is now written as

$$\nabla^2 \varphi = u \frac{\partial \varphi}{\partial r} + \frac{v}{r} \frac{\partial \varphi}{\partial \theta} - \exp(b\varphi)(1 - k(\varphi + 1)) + \frac{\partial \varphi}{\partial t}. \qquad (18)$$

Boundary conditions remain the same similar to the previous case. The solution of this hydrodynamic system is shown in Fig. 3 where a dependence of the Frank-Kamenetskii parameter on the maximum temperature in the working area of the cylinder is presented for various values of the energy characteristic parameter of reverse processes.

One can readily see from Fig. 3, that the account of return processes clearly increases the critical values of the parameters.

The major convection effect is found in the range of thermal explosion, i.e. in the area of the first maximum of the function b (φ). Curves in Fig. 3 practically coincide at low and high temperatures. Though the maximum change of the parameter b in the area of thermal explosion is about 20%, the effect of this change becomes significant because the factor of b stands in the exponential function and $b_w\varphi \sim 10$. Therefore, even a relatively small change of b may lead to a substantial change of the energy contribution. One notes that in fact the parameter of heat dissipation k cannot be large and therefore the thermal explosion occurs during the increase of the energy contribution.

Fig. 3 is supplemented by Fig. 4 where the process of thermal explosion in the $b - \varphi$ plane is schematically represented. At small k ($k \leqslant 0.4$) the function b (φ) has two extrema. The thermal explosion occurs when the parameter b reaches the value a_1, and as a result the system moves from the point a_1 to the point a_2 which results in a sharp temperature rise. However, this feature vanishes with increasing k as the curve $c_2a_2a_1c_1$ becomes monotonous. In this case there is no thermal explosion in the system. It should also be noted that for a monotonous curve the section a_1c_1 corresponds to an unstable mode and consequently the corresponding values of b are not realized. When the system moves from the point a_2 to the point c_1 the transition occurs along the high-temperature branch,

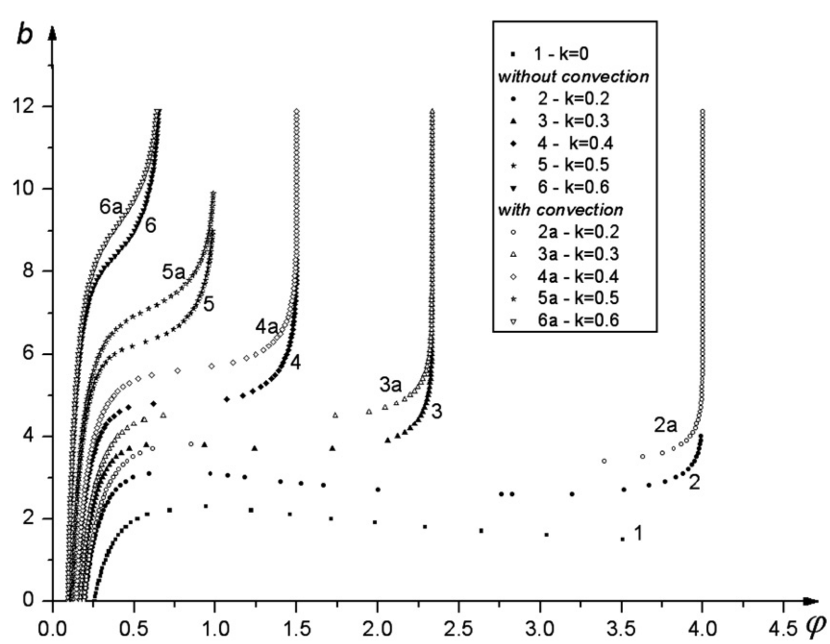

Fig. 3. Frank-Kamenetskii parameter as a function of the maximum temperature in the cylinder for different values of the energy parameter k of reverse processes.

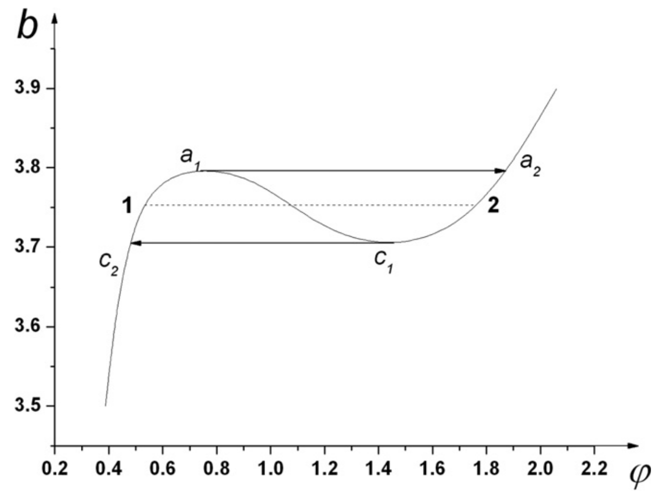

Fig. 4. The sketch of the system behavior at the thermal explosion with reverse processes. The transition $a_1 \to a_2$ corresponds to the thermal explosion. The transition $c_1 \to c_2$ appears at the decrease of the energy intake. Points 1 and 2 correspond to the two steady states corresponding to the same values of the parameter of b.

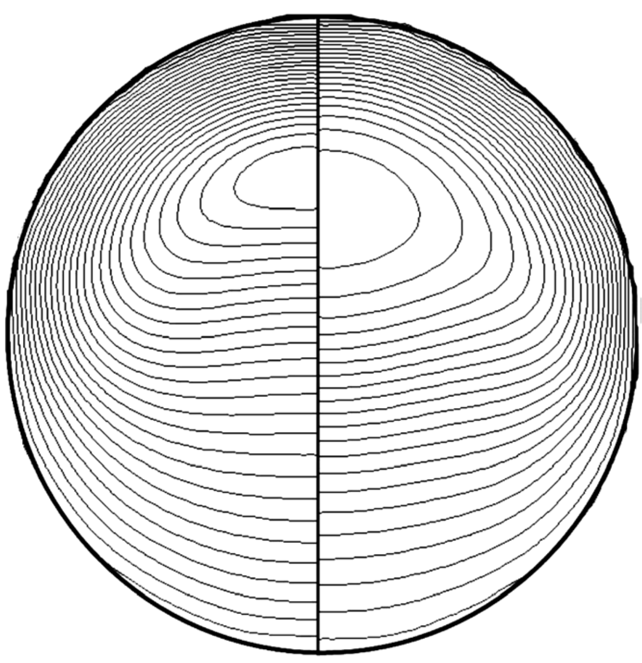

Fig. 5. Isotherms at the points 1 and 2 (see Fig. 4). Left hand side corresponds to the point 1 (low translational temperature), right hand side corresponds to the point 2 (high translational temperature). The ratio between maximum temperatures at the point 2 to 1 is equal to 4.

and from the point c_1 the system spontaneously jumps to the point c_2 with a sharp decrease of temperature. Here one finds an original "hysteresis" which has been observed experimentally in a discharge.

Points 1 and 2 in Fig. 4 correspond to the two different states of the system which correspond to the same external conditions. At the point 2 the temperature is significantly larger than at the point 1. Isotherms and streamlines for

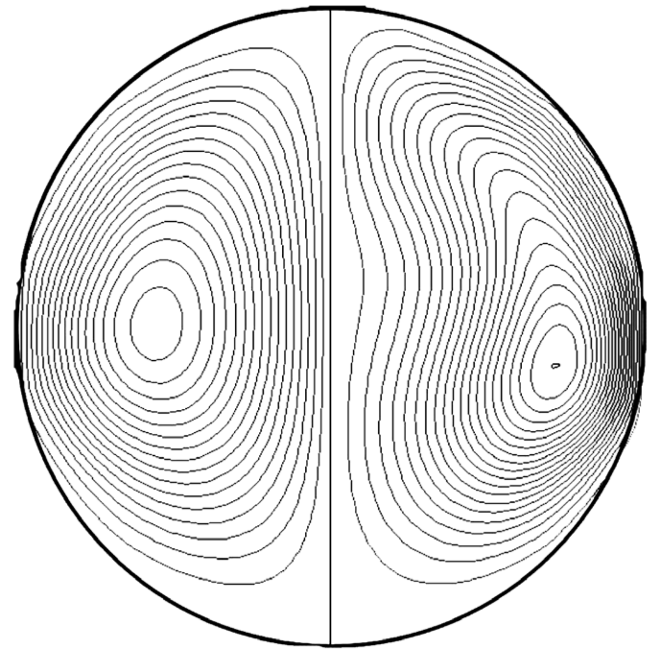

Fig. 6. Streamlines at the point 1 and 2 (see Fig. 4). Left hand side corresponds to the point 1 (low translational temperature), right hand side corresponds to the point 2 (high translational temperature). The ratio between maximum velocities at the point 2 to 1 is equal to 1.24.

these two cases are shown in Figs. 5 and 6. One can readily see that there are no fundamental differences in the flow structure. However, temperature values in the hot gas are significantly larger although the velocities are the same.

4. Conclusions

1. The problem of thermal explosion with natural convection and the reverse endothermic reactions has been solved.
2. It has been shown that convection particularly strongly influences the parameters of the system in the area of thermal explosion. This conclusion justifies the necessity to take the convection into account in any theoretical consideration of gas lasers and real discharges.

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